
Three-Level Calculations

Keith Corzine

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This Mathcad sheet plots waveforms for a three-level inverter. Individual transistor and diode waveforms are created for calculation of switching and conduction losses as well as peak and average currents.

Inverter ratings

$v_{dc} := 650 \cdot V$ $\quad i_s := 100A$ $\quad pf := 0.8$ $\quad f := 60 \cdot Hz$

$m := 0.95$ $\quad f_{sw} := 5 \cdot kHz$

transistor ratings

$v_{IGBT} := \frac{v_{dc}}{2}$ $\quad v_{IGBT} = 325\ V$

$i_{IGBT} := \sqrt{2} \cdot i_s$ $\quad i_{IGBT} = 141\ A$

phase current

$\phi_i := -acos(pf)$ $\quad \left(\frac{180}{\pi}\right) \cdot \phi_i = -36.87$

$i_{as}(\theta) := \sqrt{2} \cdot i_s \cdot cos(\theta + \phi_i)$

$I_a(\theta) := if(i_{as}(\theta) > 0, 1, 0)$

power factor check

$\tau := \frac{-tan(\phi_i)}{2 \cdot \pi \cdot f}$ $\quad \tau = 1.989 \times 10^{-3} s$

$T_{sw} := \frac{1}{f_{sw}}$ $\quad T_{sw} = 200 \times 10^{-6} s$

$\frac{\tau}{T_{sw}} = 9.947$ <--- ratio of load time constant to switching period
This should be significant for accuracy

duty cycle

$$d_{am}(\theta) := 1 + m\cdot\cos(\theta)$$

triangle waveforms

$$tr_1(\theta) := \frac{1}{2} + \frac{4}{\pi^2}\cdot\left[\sum_{n=1}^{5} \frac{\cos\left[(2\cdot n - 1)\cdot\theta\cdot\frac{f_{sw}}{f}\right]}{(2\cdot n - 1)^2}\right]$$

$$tr_2(\theta) := \frac{3}{2} + \frac{-4}{\pi^2}\cdot\left[\sum_{n=1}^{5} \frac{\cos\left[(2\cdot n - 1)\cdot\theta\cdot\frac{f_{sw}}{f}\right]}{(2\cdot n - 1)^2}\right]$$

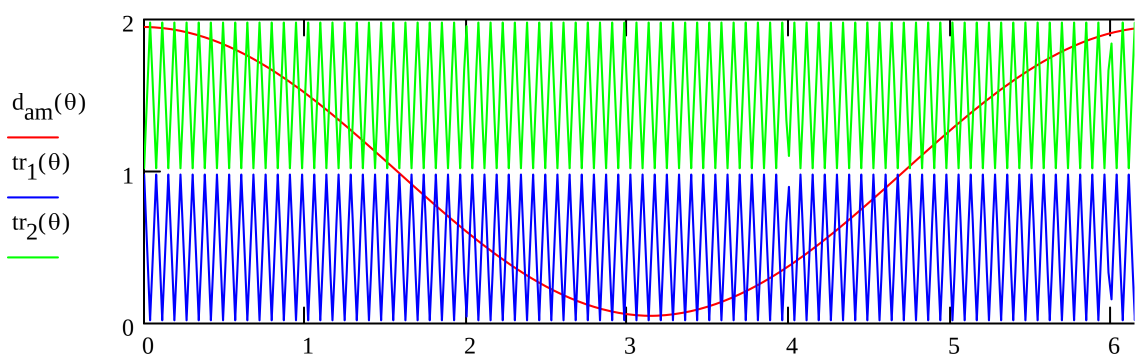
switching functions

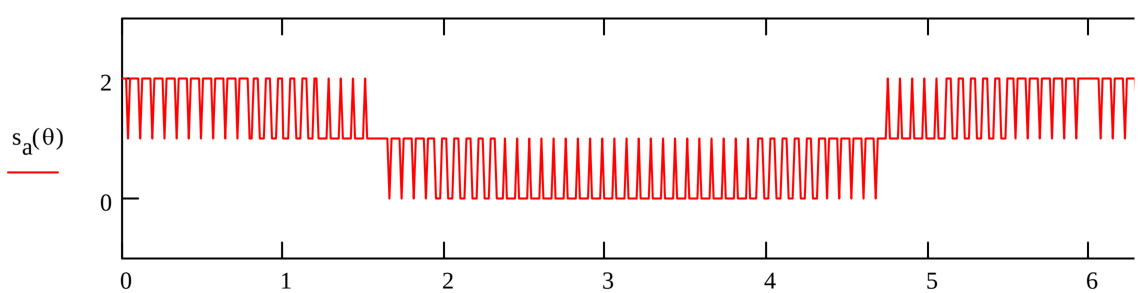
$$s_a(\theta) := \text{if}\left(d_{am}(\theta) > tr_1(\theta), 1, 0\right) + \text{if}\left(d_{am}(\theta) > tr_2(\theta), 1, 0\right)$$

$$s_{a0}(\theta) := \text{if}\left(s_a(\theta) = 0, 1, 0\right)$$

$$s_{a1}(\theta) := \text{if}\left(s_a(\theta) = 1, 1, 0\right)$$

$$s_{a2}(\theta) := \text{if}\left(s_a(\theta) = 2, 1, 0\right)$$





line-to-ground voltage

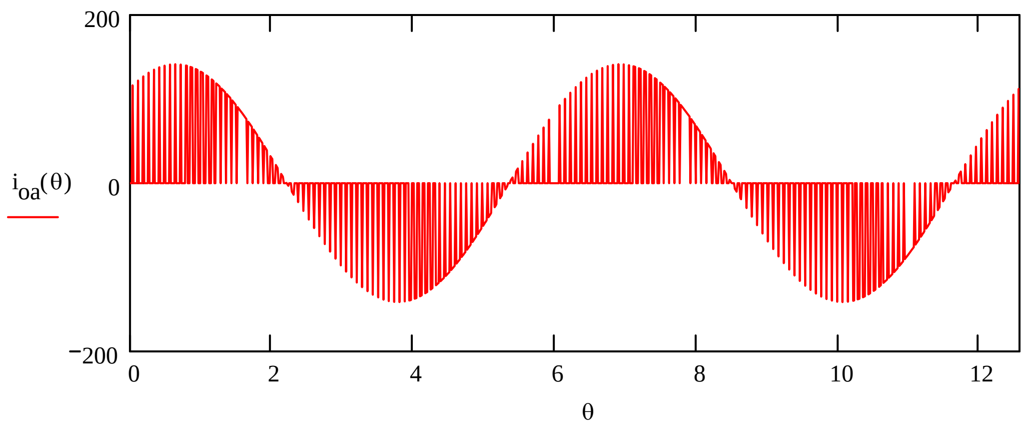
$$v_{ag}(\theta) := \frac{v_{dc}}{2} \cdot s_a(\theta)$$

neutral-point current (a-phase contribution)

$$i_{oa}(\theta) := s_{a1}(\theta) \cdot i_{as}(\theta)$$

$$i_{c2a}(\theta) := s_{a2}(\theta) \cdot i_{as}(\theta)$$

$$i_{c1a}(\theta) := s_{a0}(\theta) \cdot i_{as}(\theta)$$



$$t_{min} := 20 \cdot 10^{-6} \cdot sec$$

$$n := \frac{2}{6 \cdot f \cdot t_{min}} \qquad n = 278$$

$$i := 0, 1 .. (n-1)$$

$$i_{ca\_rms} := \sqrt{\frac{3}{\pi \cdot 2 \cdot n} \cdot \sum_i \left(i_{c1a}\left(\frac{i}{n-1} \cdot \frac{\pi \cdot 2}{3} \right) \right)^2} \qquad i_{ca\_rms} = 7.1\,A$$

transistor and diode votlages

$$v_{Ta1}(\theta) := \frac{v_{dc}}{2} \cdot (1 - s_{a2}(\theta))$$

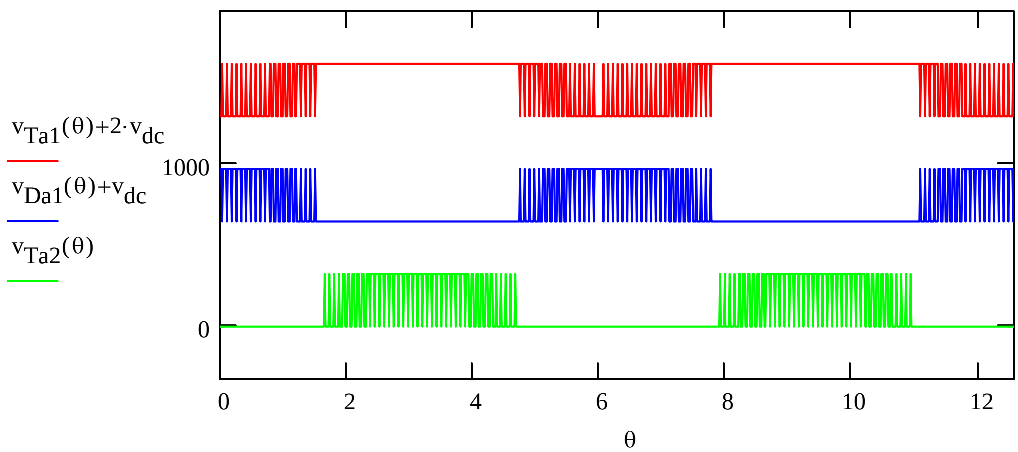
$$v_{Da1}(\theta) := \frac{v_{dc}}{2} \cdot s_{a2}(\theta)$$

$$v_{Ta2}(\theta) := \frac{v_{dc}}{2} \cdot s_{a0}(\theta)$$

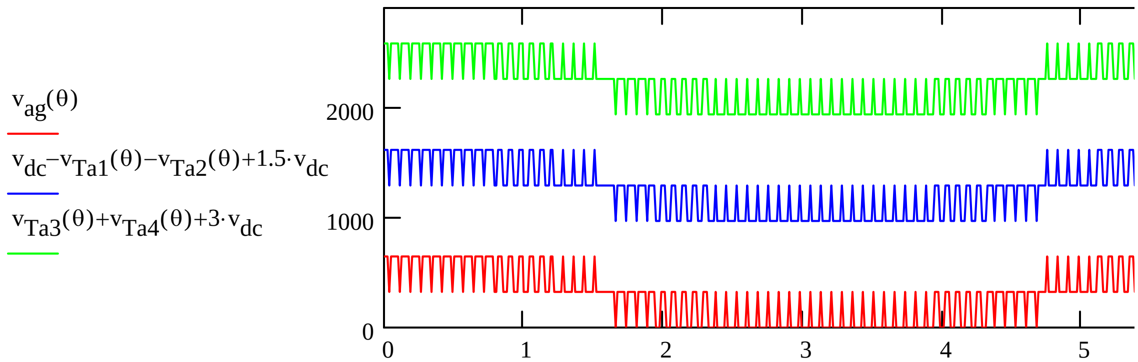
$$v_{Ta3}(\theta) := \frac{v_{dc}}{2} \cdot s_{a2}(\theta)$$

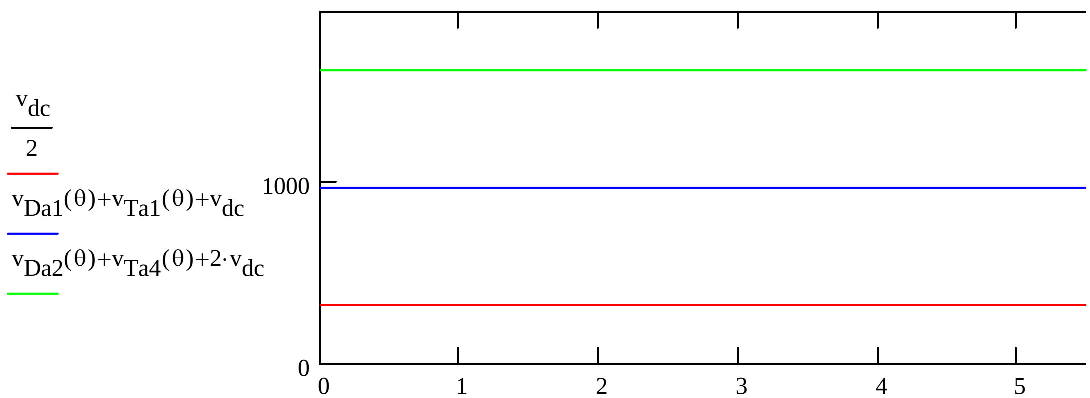
$$v_{Da2}(\theta) := \frac{v_{dc}}{2} \cdot s_{a0}(\theta)$$

$$v_{Ta4}(\theta) := \frac{v_{dc}}{2} \cdot (1 - s_{a0}(\theta))$$



diode and transistor voltage checksum





$$\frac{v_{dc}}{2} = 325\,\text{V}$$

$$v_{Da1}(0) + v_{Ta1}(0) = 325\,\text{V}$$

$$v_{Da2}(0) + v_{Ta4}(0) = 325\,\text{V}$$

transistor and diode currents

$$i_{Ta1}(\theta) := i_{as}(\theta)\cdot I_a(\theta)\cdot s_{a2}(\theta)$$

$$i_{Ta1FD}(\theta) := -i_{as}(\theta)\cdot\left(1 - I_a(\theta)\right)\cdot s_{a2}(\theta)$$

$$i_{Da1}(\theta) := i_{as}(\theta)\cdot I_a(\theta)\cdot s_{a1}(\theta)$$

$$i_{Ta2}(\theta) := i_{as}(\theta)\cdot I_a(\theta)\cdot\left(1 - s_{a0}(\theta)\right)$$

$$i_{Ta2FD}(\theta) := -i_{as}(\theta)\cdot\left(1 - I_a(\theta)\right)\cdot s_{a2}(\theta)$$

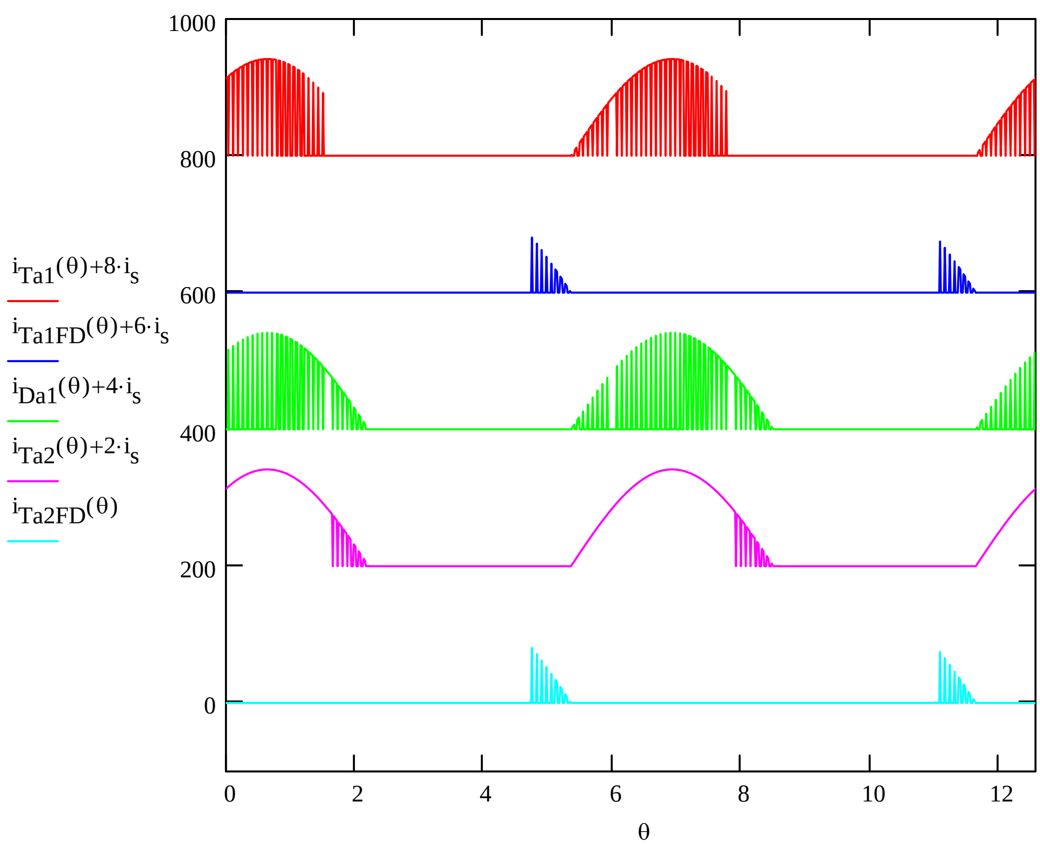
$$i_{Ta3}(\theta) := -i_{as}(\theta)\cdot\left(1 - I_a(\theta)\right)\cdot\left(1 - s_{a2}(\theta)\right)$$

$$i_{Ta3FD}(\theta) := i_{as}(\theta)\cdot I_a(\theta)\cdot s_{a0}(\theta)$$

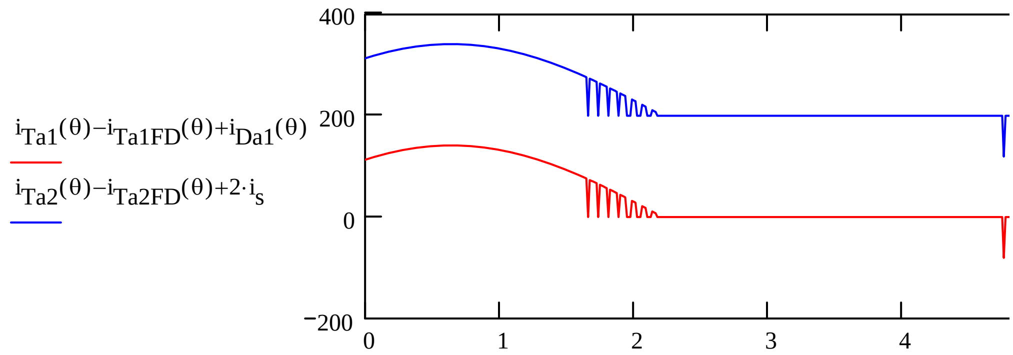
$$i_{Da2}(\theta) := -i_{as}(\theta)\cdot\left(1 - I_a(\theta)\right)\cdot s_{a1}(\theta)$$

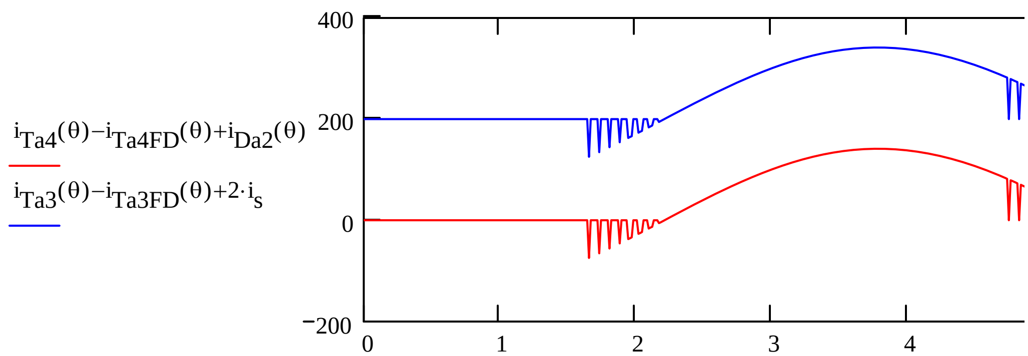
$$i_{Ta4}(\theta) := -i_{as}(\theta)\cdot\left(1 - I_a(\theta)\right)\cdot s_{a0}(\theta)$$

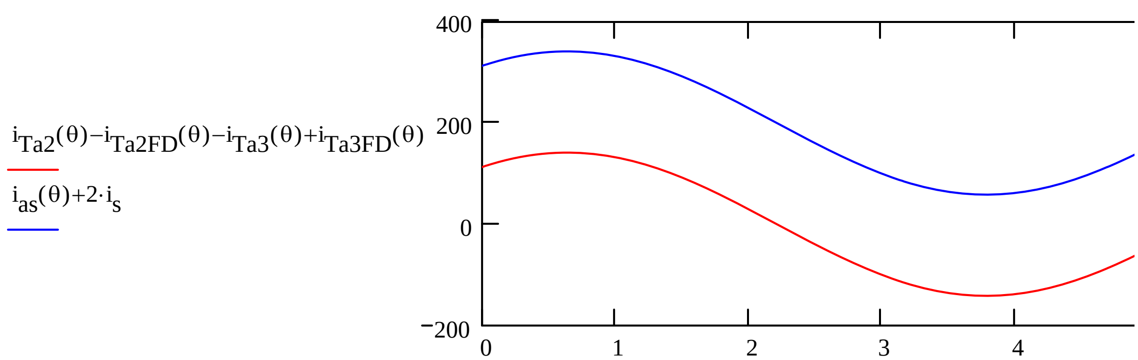
$$i_{Ta4FD}(\theta) := i_{as}(\theta)\cdot I_a(\theta)\cdot s_{a0}(\theta)$$

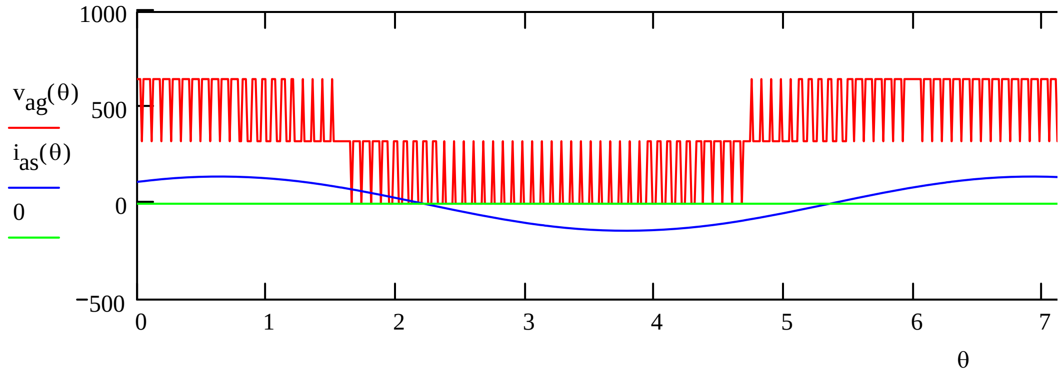


transistor and diode current checksum

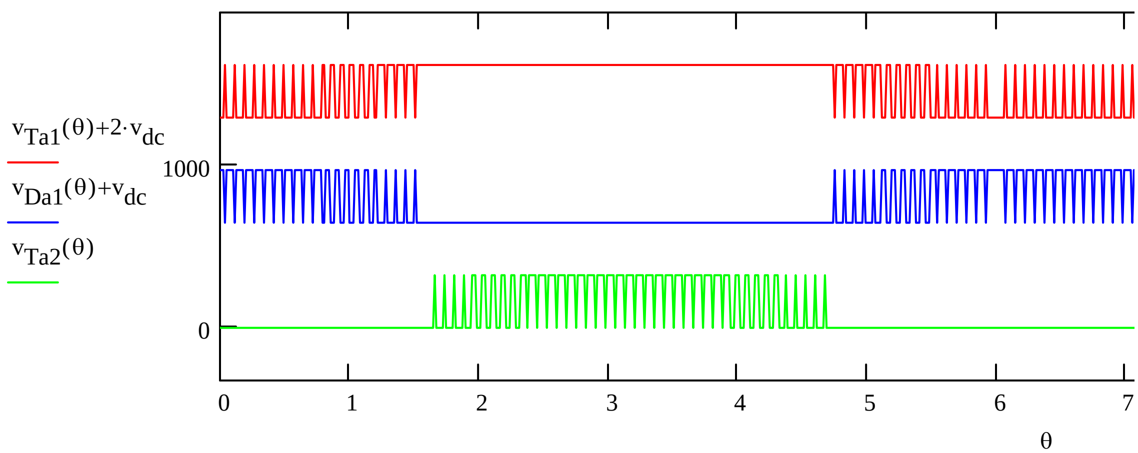




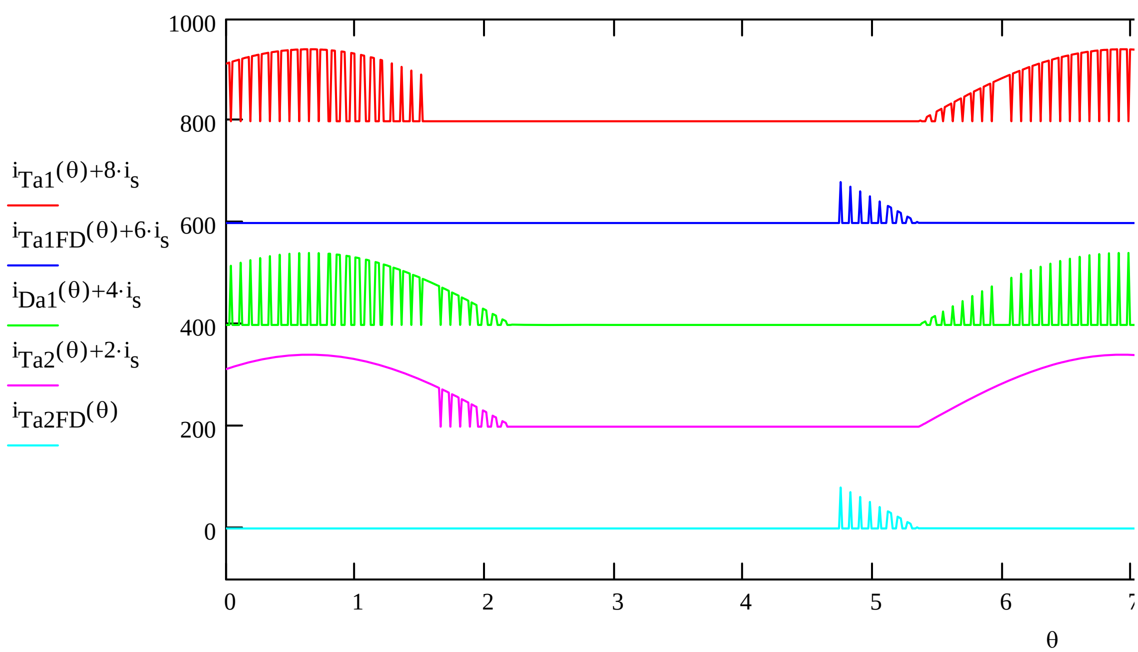




$v_{ag}(\theta)$
$i_{as}(\theta)$
0
1000
500
0
−500
0 1 2 3 4 5 6 7
θ



$v_{Ta1}(\theta)+2\cdot v_{dc}$
$v_{Da1}(\theta)+v_{dc}$
$v_{Ta2}(\theta)$
1000
0
0 1 2 3 4 5 6 7
θ



$i_{Ta1}(\theta)+8\cdot i_s$
$i_{Ta1FD}(\theta)+6\cdot i_s$
$i_{Da1}(\theta)+4\cdot i_s$
$i_{Ta2}(\theta)+2\cdot i_s$
$i_{Ta2FD}(\theta)$
1000
800
600
400
200
0
0 1 2 3 4 5 6 7
θ

average currents and conduction losses

$V_Q := 2 \cdot volt \qquad V_D := 1.5 \cdot volt$

$t_{min} := 20 \cdot 10^{-6} \cdot sec$

$n := \frac{1}{f \cdot t_{min}} \qquad n = 833$

$i := 0, 1 .. (n - 1)$

$i_{Ta1\_avg} := \left(\frac{1}{n-1}\right) \cdot \sum_i i_{Ta1}\left(\frac{i \cdot 2 \cdot \pi}{n-1}\right) \qquad i_{Ta1\_avg} = 27.62\,A$

$i_{Ta1FD\_avg} := \left(\frac{1}{n-1}\right) \cdot \sum_i i_{Ta1FD}\left(\frac{i \cdot 2 \cdot \pi}{n-1}\right) \qquad i_{Ta1FD\_avg} = 0.93\,A$

$i_{Da1\_avg} := \left(\frac{1}{n-1}\right) \cdot \sum_i i_{Da1}\left(\frac{i \cdot 2 \cdot \pi}{n-1}\right) \qquad i_{Da1\_avg} = 16.54\,A$

$i_{Ta2\_avg} := \left(\frac{1}{n-1}\right) \cdot \sum_i i_{Ta2}\left(\frac{i \cdot 2 \cdot \pi}{n-1}\right) \qquad i_{Ta2\_avg} = 44.15\,A$

$i_{Ta2FD\_avg} := \left(\frac{1}{n-1}\right) \cdot \sum_i i_{Ta2FD}\left(\frac{i \cdot 2 \cdot \pi}{n-1}\right) \qquad i_{Ta2FD\_avg} = 0.93\,A$

$P_{cond\_Ta1} := i_{Ta1\_avg} \cdot V_Q \qquad P_{cond\_Ta1} = 55.23\,W$

$P_{cond\_Ta1FD} := i_{Ta1FD\_avg} \cdot V_D \qquad P_{cond\_Ta1FD} = 1.4\,W$

$P_{cond\_Da1} := i_{Da1\_avg} \cdot V_D \qquad P_{cond\_Da1} = 24.81\,W$

$P_{cond\_Ta2} := i_{Ta2\_avg} \cdot V_Q \qquad P_{cond\_Ta2} = 88.31\,W$

$P_{cond\_Ta2FD} := i_{Ta2FD\_avg} \cdot V_D \qquad P_{cond\_Ta2FD} = 1.4\,W$

$P_{cond} := 6 \cdot P_{cond\_Ta1} + 6 \cdot P_{cond\_Ta1FD} + 6 \cdot P_{cond\_Da1} \cdots$
$+ 6 \cdot P_{cond\_Ta2} + 6 \cdot P_{cond\_Ta2FD}$

$\boxed{P_{cond} = 1027\,W}$

switching losses

$E_{on} := 0.13 \cdot watt \cdot sec$ $\quad E_{off} := 0.13 \cdot watt \cdot sec$ $\quad v_E := 1200 \cdot volt$ $\quad i_E := 300 \cdot amp$

$E_{fr} := 0 \cdot watt \cdot sec$ $\quad E_{rr} := 0.13 \cdot watt \cdot sec$

$E_t := 0.5 \cdot (E_{on} + E_{off})$ $\quad E_t = 0.13\,J$

$E_d := 0.5 \cdot (E_{fr} + E_{rr})$ $\quad E_d = 0.065\,J$

$t_{min} := 5 \cdot 10^{-6} \cdot sec$

$n := \frac{1}{f \cdot t_{min}}$ $\quad n = 3333$

$i := 0, 1 .. (n-1)$

$\Delta\theta := \frac{2 \cdot \pi}{n-1}$

$$s_{Ta1_i} := \text{if}\left[i_{Ta1}(i \cdot \Delta\theta) = 0 \wedge i_{Ta1}[(i+1) \cdot \Delta\theta] \neq 0, 1, 0\right] \ldots + \text{if}\left[i_{Ta1}(i \cdot \Delta\theta) \neq 0 \wedge i_{Ta1}[(i+1) \cdot \Delta\theta] = 0, 1, 0\right]$$

$j := \text{match}(1, s_{Ta1})$

$m := \text{rows}(j)$

$k := 0, 1 .. (m-1)$

$$P_{sw\_Ta1} := \frac{f \cdot E_t}{v_E \cdot i_E} \cdot \sum_k \left[v_{Ta1}(j_k \cdot \Delta\theta) \cdot i_{Ta1}\left[(j_k + 1) \cdot \Delta\theta\right] \ldots + v_{Ta1}\left[(j_k + 1) \cdot \Delta\theta\right] \cdot i_{Ta1}(j_k \cdot \Delta\theta) \right]$$

$P_{sw\_Ta1} = 46.6\,W$

$$s_{Ta1FD_i} := \text{if}\left[i_{Ta1FD}(i \cdot \Delta\theta) = 0 \wedge i_{Ta1FD}[(i+1) \cdot \Delta\theta] \neq 0, 1, 0\right] \ldots + \text{if}\left[i_{Ta1FD}(i \cdot \Delta\theta) \neq 0 \wedge i_{Ta1FD}[(i+1) \cdot \Delta\theta] = 0, 1, 0\right]$$

$j := \text{match}(1, s_{Ta1FD})$

$m := \text{rows}(j)$

$k := 0, 1 .. (m-1)$

$$P_{sw\_Ta1FD} := \frac{f \cdot E_d}{v_E \cdot i_E} \cdot \sum_k \left[v_{Ta1}(j_k \cdot \Delta\theta) \cdot i_{Ta1FD}\left[(j_k + 1) \cdot \Delta\theta\right] \ldots + v_{Ta1}\left[(j_k + 1) \cdot \Delta\theta\right] \cdot i_{Ta1FD}(j_k \cdot \Delta\theta) \right]$$

$P_{sw\_Ta1FD} = 2.6\,W$

switching losses (continued)

$$s_{Da1_i} := \text{if}\left[i_{Da1}(i\cdot\Delta\theta) = 0 \wedge i_{Da1}[(i+1)\cdot\Delta\theta] \neq 0, 1, 0\right] \ldots$$
$$+ \text{if}\left[i_{Da1}(i\cdot\Delta\theta) \neq 0 \wedge i_{Da1}[(i+1)\cdot\Delta\theta] = 0, 1, 0\right]$$

$$j := \text{match}(1, s_{Da1})$$

$$m := \text{rows}(j)$$

$$k := 0, 1 .. (m-1)$$

$$P_{sw\_Da1} := \frac{f\cdot E_d}{v_E\cdot i_E}\cdot\sum_k \begin{bmatrix} v_{Da1}(j_k\cdot\Delta\theta)\cdot i_{Da1}\left[\left(j_k+1\right)\cdot\Delta\theta\right] \ldots \\ + v_{Da1}\left[\left(j_k+1\right)\cdot\Delta\theta\right]\cdot i_{Da1}(j_k\cdot\Delta\theta) \end{bmatrix}$$

$$P_{sw\_Da1} = 23.3\,W$$

$$s_{Ta2_i} := \text{if}\left[i_{Ta2}(i\cdot\Delta\theta) = 0 \wedge i_{Ta2}[(i+1)\cdot\Delta\theta] \neq 0, 1, 0\right] \ldots$$
$$+ \text{if}\left[i_{Ta2}(i\cdot\Delta\theta) \neq 0 \wedge i_{Ta2}[(i+1)\cdot\Delta\theta] = 0, 1, 0\right]$$

$$j := \text{match}(1, s_{Ta2})$$

$$m := \text{rows}(j)$$

$$k := 0, 1 .. (m-1)$$

$$P_{sw\_Ta2} := \frac{f\cdot E_t}{v_E\cdot i_E}\cdot\sum_k \begin{bmatrix} v_{Ta2}(j_k\cdot\Delta\theta)\cdot i_{Ta2}\left[\left(j_k+1\right)\cdot\Delta\theta\right] \ldots \\ + v_{Ta2}\left[\left(j_k+1\right)\cdot\Delta\theta\right]\cdot i_{Ta2}(j_k\cdot\Delta\theta) \end{bmatrix}$$

$$P_{sw\_Ta2} = 4.5\,W$$

$$s_{Ta2FD_i} := \text{if}\left[i_{Ta2FD}(i\cdot\Delta\theta) = 0 \wedge i_{Ta2FD}[(i+1)\cdot\Delta\theta] \neq 0, 1, 0\right] \ldots$$
$$+ \text{if}\left[i_{Ta2FD}(i\cdot\Delta\theta) \neq 0 \wedge i_{Ta2FD}[(i+1)\cdot\Delta\theta] = 0, 1, 0\right]$$

$$j := \text{match}(1, s_{Ta2FD})$$

$$m := \text{rows}(j)$$

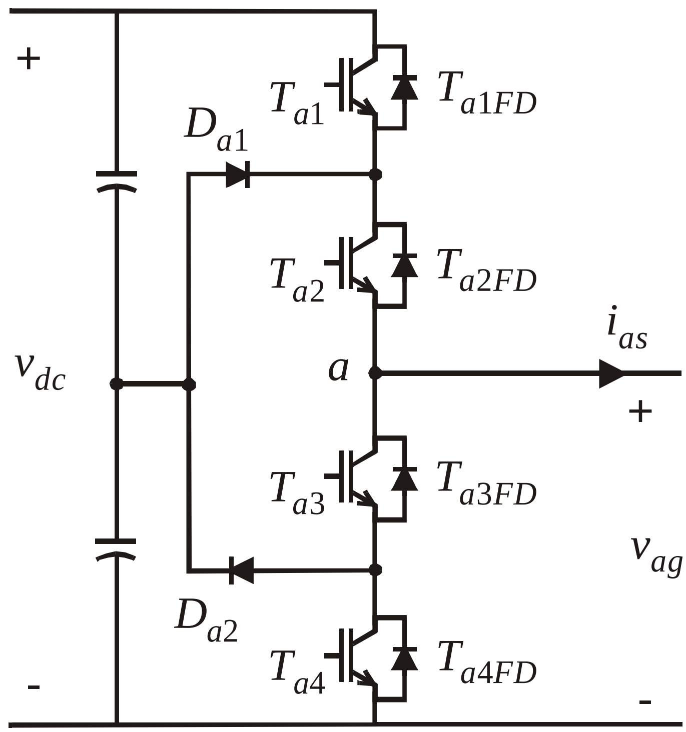
$$k := 0, 1 .. (m-1)$$

$$P_{sw\_Ta2FD} := \frac{f\cdot E_d}{v_E\cdot i_E}\cdot\sum_k \begin{bmatrix} v_{Ta2}(j_k\cdot\Delta\theta)\cdot i_{Ta2FD}\left[\left(j_k+1\right)\cdot\Delta\theta\right] \ldots \\ + v_{Ta2}\left[\left(j_k+1\right)\cdot\Delta\theta\right]\cdot i_{Ta2FD}(j_k\cdot\Delta\theta) \end{bmatrix}$$

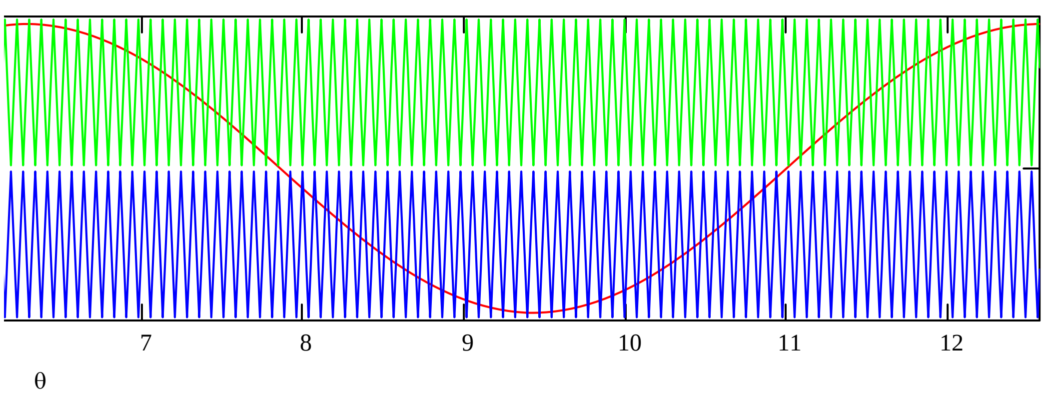
$$P_{sw\_Ta2FD} = 0\,W$$

$$P_{sw} := 6\cdot P_{sw\_Ta1} + 6\cdot P_{sw\_Ta1FD} + 6\cdot P_{sw\_Da1} \cdots$$
$$+ 6\cdot P_{sw\_Ta2} + 6\cdot P_{sw\_Ta2FD}$$

$$\boxed{P_{sw} = 462\,W}$$



Three-level diode-clamped topology (a-phase)



7
8
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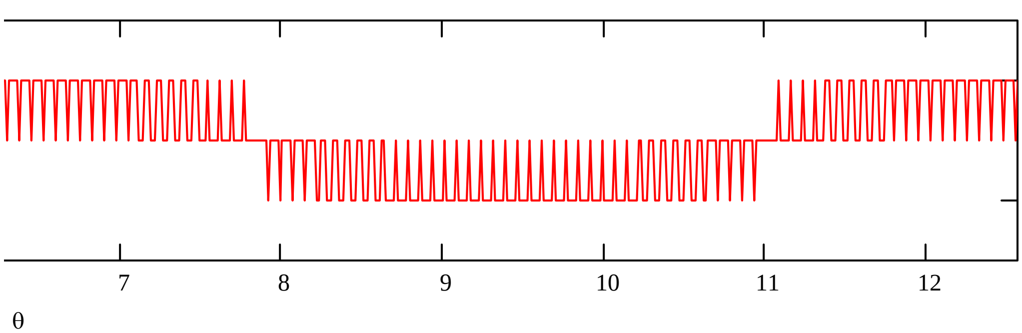
12
θ



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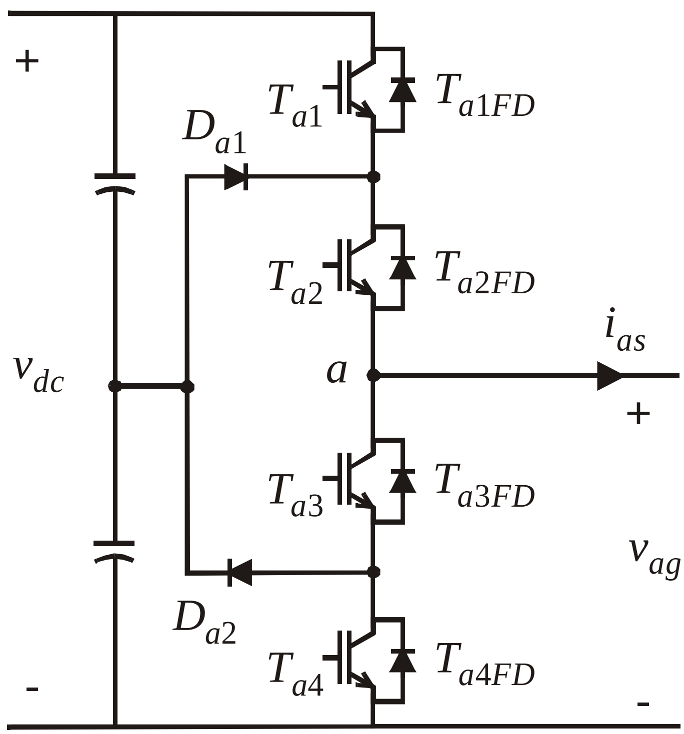
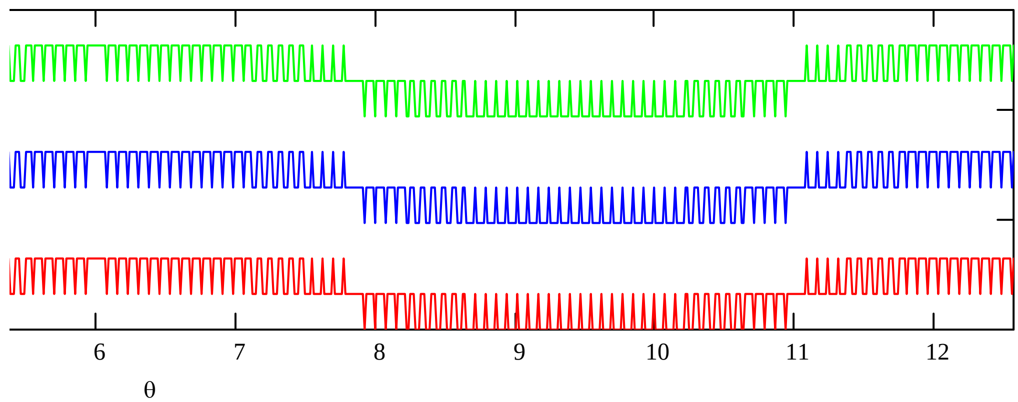


Three-level diode-clamped topology (a-phase)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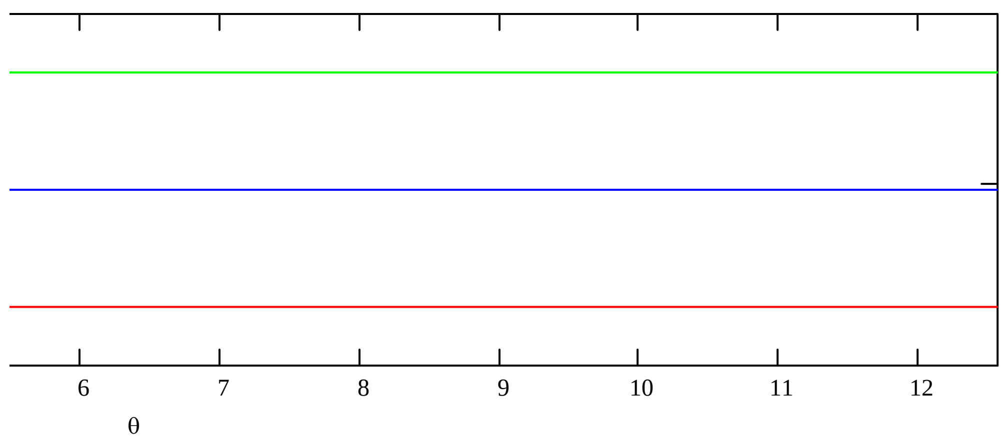
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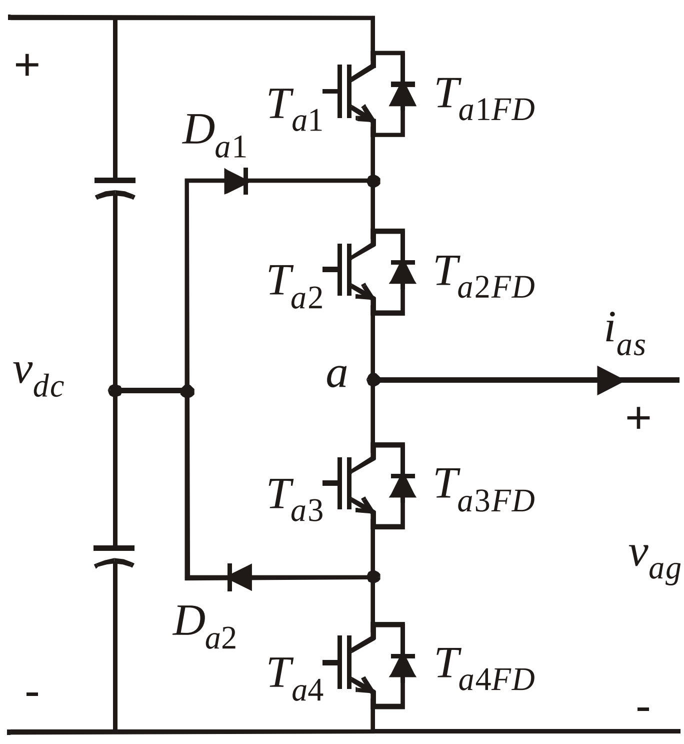
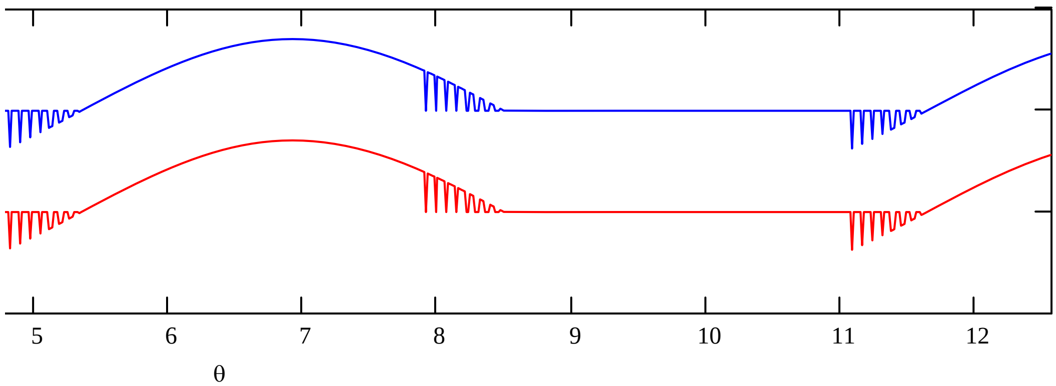
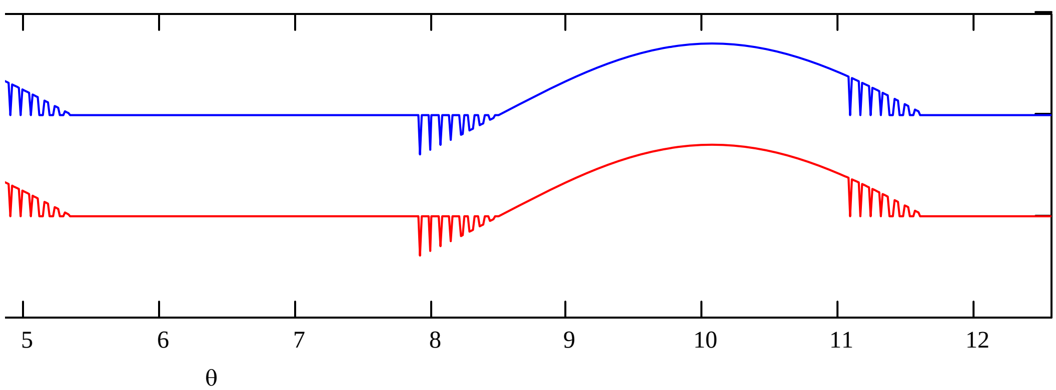


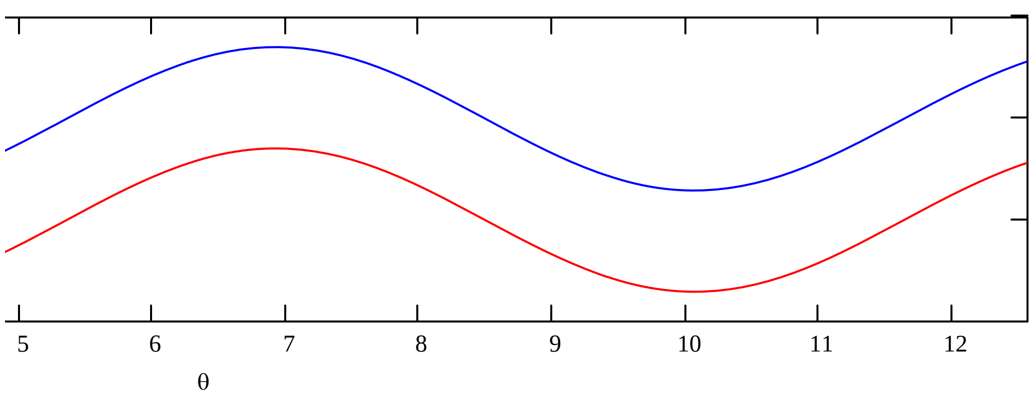
Three-level diode-clamped topology (a-phase)



5 6 7 8 9 10 11 12
θ



5 6 7 8 9 10 11 12
θ



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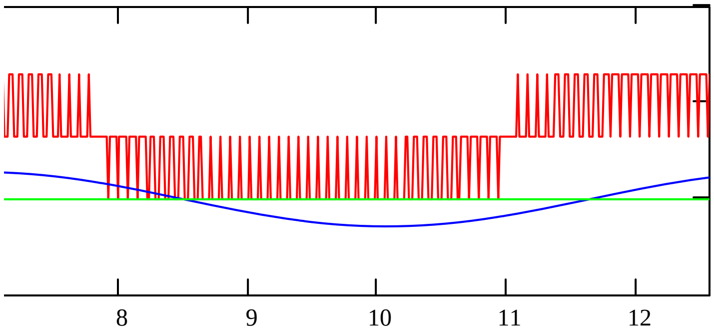
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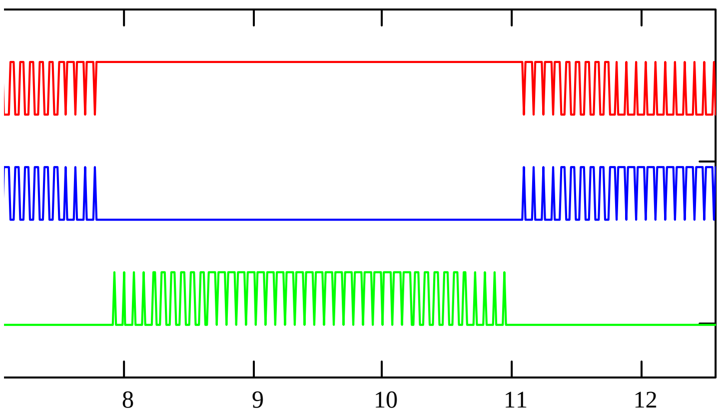
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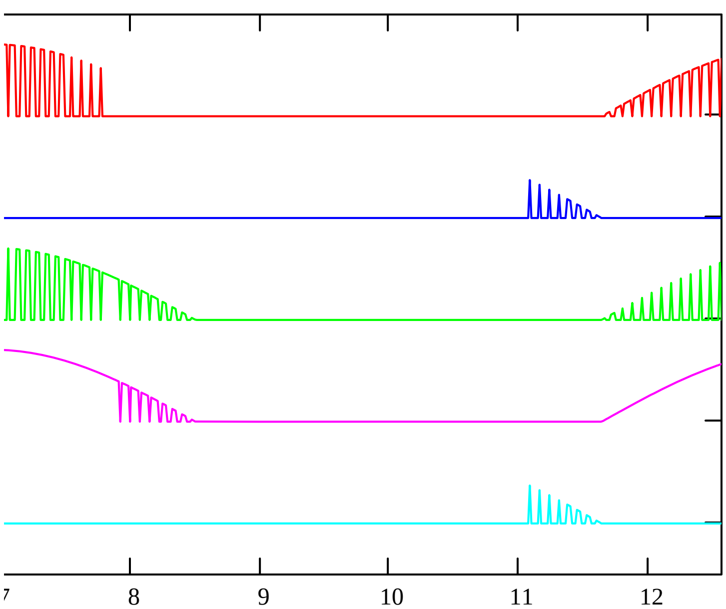
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$$i_{Ta1i_i} := i_{Ta1}\left(\frac{i \cdot 2 \cdot \pi}{n-1}\right)$$

$$i_{Ta1\_max} := \max\left(i_{Ta1i}\right)$$

$$i_{Ta1\_max} = 141\ A$$

$$i_{Ta1\_avg} := \left(\frac{1}{n-1}\right) \cdot \sum_{i} i_{Ta1i_i}$$

$$i_{Ta1\_avg} = 27.62\ A$$